Magnetic monopole mass from fundamental lengths

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With an assumption of $n = 2$ in the Dirac quantization condition, when the sequence comprised of the Bohr radius, reduced Compton wavelength and classical electron radius is extended and matched with its velocity sequence, these fundamental lengths are mirrored with a corresponding set of fundamental monopole lengths with the speed of light as the axis of symmetry. Monopole charge is $g = 137ec$ and mass is found to be 1.45×10^{-3} eV/c^2. This result may fit with the extended relativity (tachyon) theory of Recami with a modification of the proposed charge quantization and electromagnetic coupling constant.

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I. MONOPOLES

Dirac introduced the idea of electric charge quantization using the quantization condition[1,2]

$$g = \frac{n\hbar c}{2e} = ng_D \qquad n = (0, \pm1, \pm2, \pm3, ...), \tag{1}$$

which set the ground rules for magnetic charge. Since this relation was published, many studies[3,4] have set out to understand and find magnetic monopoles[5], hereafter referred to as *monopoles*.

Three potential detections of the particle stand unconfirmed and so the monopole remains unobserved. The search continues, most notably at CERN[6], because the implications of actually finding the monopole are extraordinary.

Even though no real prediction of monopole mass has ever been made, attempts have been made to estimate it.

An oft-quoted estimate is based on the assumption that the classical electron radius, r_0, is the same for the electron and monopole. Given this, the monopole mass can be estimated as $m_m = g^2 m_e/(ec)^2 \simeq 4692m_e \simeq 2.4$ GeV/c^2.

Graf, *et.al*[7] measured the anomalous magnetic moment of the muon to give a lower limit on a Dirac monopole mass of 120 GeV/c^2.

The D0 collaboration[8] set lower mass limits of 610, 870, and 1580 GeV/c^2 for Dirac monopoles of spin 0, 1/2 or 1.

In grand unified theories, the lower limit of monopole mass is about 10^{15}GeV/c^2.

II. HIERARCHIES OF LENGTHS

Using the subscript 'e' for electron, the hierarchy of lengths

$$r_{0e} < \lambdabar_{ce} < a_{0e}, \tag{2}$$

is comprised of the classical electron radius, r_{0e}, the reduced[9] Compton wavelength, $\lambdabar_{ce}$, and the Bohr radius, a_{0e}.

Using eq. (1) with $n = 1$, Datta[10] determined the fundamental monopole lengths and showed that the hierarchy

$$a_{0m} < \lambdabar_{cm} < r_{0m}, \tag{3}$$

(using subscripts of 'm' for monopole), exists for monopoles, that it makes monopoles inherently relativistic and that atom-like bound states of monopoles would require superluminal ground state orbital speeds.

Datta rejected the required faster-than-light velocities for monopole atoms as unphysical and therefore concluded that monopoles are unlikely to exist.

As an alternative, the paradox of monopole fundamental lengths is examined from the perspective of *extended relativity* where the possibility of symmetric hierarchies (2) and (3) are admitted.

III. EXTENDED RELATIVITY

Sudarshan[11,12], Feinberg[13,14], Recami[15,16] and others advanced ideas that superluminal velocities are not necessarily unphysical and formed a framework for investigation of faster-than-light particles called *tachyons*.

Parker[17] originally suggested that when special relativity is extended to superluminal velocities, charged tachyons should look like monopoles to subluminal observers.

Recami and Mignani developed Parker's extended relativity and the idea of tachyons naturally appearing to normal subluminal observers as magnetically charged particles due to special relativity.[15,16,18–22] Using a symmetry argument, this model notably proposed a monopole charge of $g = -ec$, which is much lower than the Dirac value of $g = 68.5ec$ or the Schwinger value of $g = 137ec$.

Using a value of $n = 2$ (*i.e.* $g = 137ec$) in (1) as opposed to $n = 1$ used in[10], the values of the ground state orbital velocity of the electron, $v_{0e} = c\alpha$, and the monopole, $v_{0m} = c/\alpha$, become exactly symmetric with the speed of light as the axis of symmetry. These velocities in turn correspond to a_{0e} and a_{0m}, the Bohr radii for electrons and monopoles. Thus Schwinger's relation,[23]

$$g = \frac{n\hbar c}{e} \qquad n = (0, \pm 1, \pm 2, \pm 3, ...), \tag{4}$$

(with $n = 1$) correlates with extended relativity.

IV. EXTENDING THE SEQUENCE

The reduced Compton wavelength

$$ƛ_{ce} = \frac{\hbar}{m_e c} \tag{5}$$

generates an extended sequence (in terms of the electron mass, m_e) when multiplied with α^n including both hierarchies (2) and (3)

$$ƛ_{ce}\alpha^n = \frac{\hbar\alpha^n}{m_e c} \qquad n = (1, 0, -1, -2, -3, -4, -5), \tag{6}$$

where $\alpha = k_e e^2/\hbar c$ and $k_e \equiv (4\pi\varepsilon_0)^{-1}$. This is represented as the sequence

$$\begin{aligned} \mathbb{EM} = (&ƛ_{ce}\alpha^1, ƛ_{ce}\alpha^0, ƛ_{ce}\alpha^{-1}, ƛ_{ce}\alpha^{-2}, \\ &ƛ_{ce}\alpha^{-3}, ƛ_{ce}\alpha^{-4}, ƛ_{ce}\alpha^{-5}) \\ = (&r_{0e}, ƛ_{ce}, a_{0e}, d, a_{0m}, ƛ_{cm}, r_{0m}). \end{aligned} \tag{7}$$

The generic symbol, d for distance, is given to element $ƛ_{ce}\alpha^{-2}$ in eq. (7) followed by the elements of hierarchy (3).

TABLE I. Velocity and length sequence alignment.

velocity (m/s)			length (m)		
			$ƛ_{ce}\alpha^{-5}$	r_{0m}	1.86×10^{-2}
			$ƛ_{ce}\alpha^{-4}$	$ƛ_{cm}$	1.36×10^{-4}
4.11×10^{10}	v_{0m}	$c\alpha^{-1}$	$ƛ_{ce}\alpha^{-3}$	a_{0m}	9.93×10^{-7}
2.99×10^8	c	$c\alpha^0$	$ƛ_{ce}\alpha^{-2}$	d	7.25×10^{-9}
2.19×10^6	v_{0e}	$c\alpha^1$	$ƛ_{ce}\alpha^{-1}$	a_{0e}	5.29×10^{-11}
			$ƛ_{ce}\alpha^0$	$ƛ_{ce}$	3.86×10^{-13}
			$ƛ_{ce}\alpha^1$	r_{0e}	2.82×10^{-15}

Elements of the velocity sequence are defined by

$$\mathbb{V} = (v_{0e}, c, v_{0m}) = c\alpha^n \qquad n = (1, 0, -1). \tag{8}$$

$\mathbb{EM}$ is aligned with $\mathbb{V}$

$$\mathbb{EM} = (r_{0e},\, ƛ_{ce},\, a_{0e},\, d,\, a_{0m},\, ƛ_{cm},\, r_{0m}) \tag{9}$$
$$|$$
$$\mathbb{V} = (v_{0e},\, c,\, v_{0m}), \tag{10}$$

by matching v_{0e}, the ground state orbital velocity of the electron in hydrogen with a_{0e}, the Bohr radius of hydrogen. (See also Table I.)

V. MONOPOLE MASS

Mass of the monopole can be found by equating the Compton wavelength for the monopole $ƛ_{cm}$, *in terms of the unknown monopole mass*, to $ƛ_{ce}\alpha^{-4}$, the Compton wavelength of the monopole in *in terms of the electron mass*,

$$\frac{\hbar\alpha^{-4}}{m_e c} = \frac{\hbar}{m_m c} \tag{11}$$

$$m_m = m_e\alpha^4 = 1.45 \times 10^{-3} eV/c^2. \tag{12}$$

VI. CONCLUDING REMARKS

The extension of the well known sequence of physical constants comprised of the Bohr radius, the reduced Compton radius and the classical electron radius, when aligned with matching velocities, indicates a mirrored spacetime structure with the speed of light as the line of symmetry.

The reflected symmetry of electron and monopole length hierarchies fits with extended relativity as presented by Recami[15,16] except for the value of magnetic charge and the coupling constant.

With $g = 137ec$, the *magnetic* coupling constant, $\alpha_m = 137$. It appears possible to construct a theory of extended relativity with $g = 137ec$ and a coupling constant that inverts with velocity

$$\alpha_{\text{EM}} = \begin{cases} \alpha, & \text{if} \quad \beta < 1 \\ \alpha^{-1}, & \text{if} \quad \beta > 1 \end{cases} . \tag{13}$$

In addition, eq. (12) leads to the fine structure constant, $\alpha = (m_m/m_e)^{1/4}$, as a mass ratio.

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